High speed cornering strategy with confined contour error and vibration suppression for CNC machine tools

Article *in* CIRP Annals - Manufacturing Technology · May 2015

DOI: 10.1016/j.cirp.2015.04.102

3 authors, including:


Burak Sencer
Oregon State University


Eiji Shamoto
Nagoya University

CIRP Annals - Manufacturing Technology 64 (2015) 369–372

Contents lists available at ScienceDirect

CIRP Annals - Manufacturing Technology

journal homepage: http://ees.elsevier.com/cirp/default.asp

High speed cornering strategy with confined contour error and vibration suppression for CNC machine tools

Burak Sencer [a,*], Kosuke Ishizaki [b], Eiji Shamoto (1)[b]

[a] School of Mechanical, Industrial, and Manufacturing Engineering, Oregon State University, USA
[b] Department of Mechanical Engineering, Nagoya University, Chikusa-ku, Nagoya, Aichi, Japan

ARTICLE INFO

Keywords:
CNC
Tool-path
Interpolation

ABSTRACT

This paper presents a novel real-time trajectory generation algorithm for accurate high-speed cornering applications. Typically, reference tool-paths compromised of G01 lines are geometrically smoothed by means of arcs and splines. In this study, a kinematic corner smoothing algorithm approach is proposed where the cornering trajectory of the tool is generated through FIR (Finite Impulse Response) filtering of discontinuous axis velocity commands at segment junctions. Contouring errors at sharp corners are controlled analytically by optimally overlapping acceleration profiles of previous and present segments. Residual vibrations due to excitation of structural modes are avoided by tuning filter delays for all drives. The proposed method has been experimentally demonstrated to show significant improvement in the cycle time and accuracy of contouring Cartesian tool-paths.

1. Introduction

Computer-aided design (CAD) systems are utilized to design complex geometries based on smooth parametric splines such as NURBS or B-splines. Although direct interpolation of these curves is proven to be superior in terms of providing smoother and faster motion on modern machine [1–3], vast majority of numerical control (NC) systems accept reference tool-paths defined only by simple linear segments, so-called the G01 lines, or basic arcs.

In order to generate high-speed motion along linear segmented tool-paths, "geometric sharp-corner-smoothing" algorithms have been favoured. Sharp corner geometries are replaced with splines and consequent linear segments are blended smoothly. Acceleration continuous motion can be planned, and cornering errors due to blending can be controlled accurately [4,5]. Nevertheless, structural vibration frequencies may still be triggered due to rapidly changing velocity profile around the corner. Well-known techniques such as input shaping [6], or Finite Impulse Response (FIR) filtering [7] have been integrated to minimize residual vibrations. However, filtering introduces unavoidable delay and induces large contouring errors in multi-axis motion that must be compensated [8].

In this paper, we propose a novel online reference trajectory generation scheme for smooth interpolation of discrete linear tool-paths with confined cornering error. The proposed scheme is presented in Fig. 1. Chain of FIR filters is used to generate smooth velocity, acceleration, jerk profiles and shape frequency spectrum of the reference trajectory. FIR filtering introduces unavoidable delay at the end of motion commands. In this work, instead of

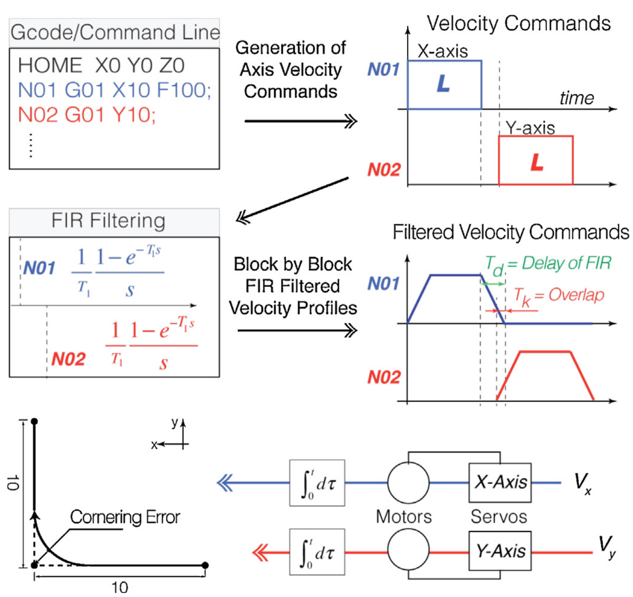

Fig. 1. Proposed corner smoothing scheme.

compensating this delay, it is made use of to synchronize drives' motion and to smoothen sharp corners with specified cornering tolerance. Fig. 1 illustrates the case where a 90° corner is smoothed by timing the interpolation of consecutive G-lines. For instance, the *X*-axis motion, commanded in the 1st G-line is elongated by the filter delay, T_d. In the proposed scheme, second G-line is interpolated before *X*-axis reaches the full stop. As a result, a sharp corner can be rounded kinematically without modifying the

* Corresponding author.
E-mail addresses: burak0307@gmail.com, burak.sencer@oregonstate.edu (B. Sencer).

http://dx.doi.org/10.1016/j.cirp.2015.04.102
0007-8506/© 2015 CIRP.

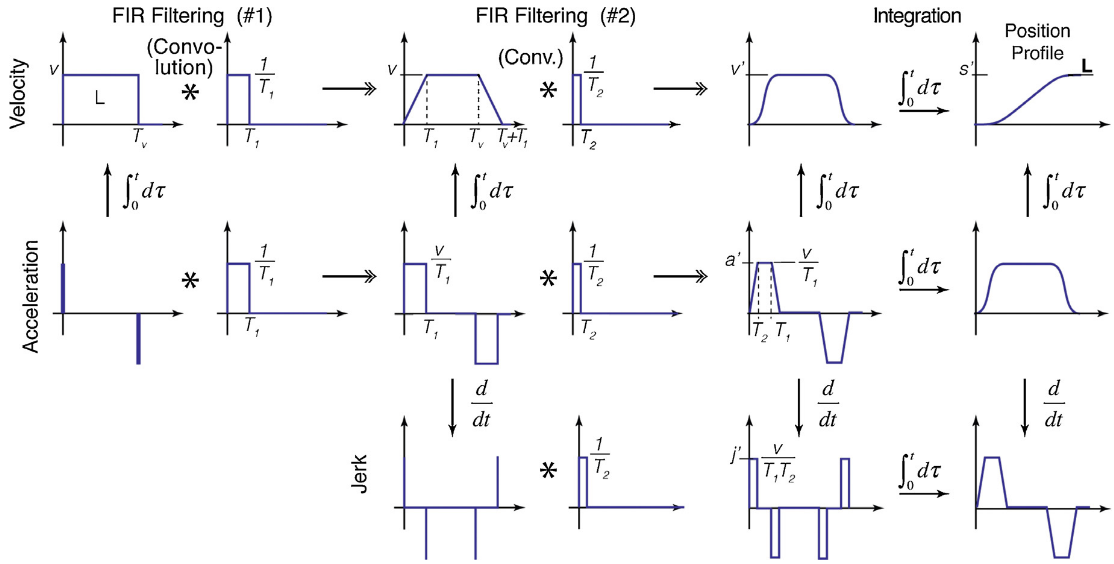

Fig. 2. Generation of smooth trajectories utilizing FIR filters.

path but by directly making use of the filter delay. The key for the proposed scheme is to accurately "time" consequent block interpolations to smooth corners with desired tolerances with respect to the filtering delay. Since the method is kinematic and simply filters the "bang-bang" type velocity profile at the corners, it delivers the fastest cycle time leading to an optimal trajectory-planning scheme. Details of the proposed multi-axis trajectory generation scheme and its experimental validation are presented.

2. FIR filtering based trajectory generation

2.1. Generation of high order kinematic profiles

Typically, "trapezoidal acceleration" or "trapezoidal jerk" based feed profiling are used for generating high-speed motion. This section introduces general methodology to generate smooth trajectories utilizing chain of n FIR filters as drafted in Fig. 2.

A 1st order FIR filter with a unity gain can be presented in Laplace(s) domain as:

$$M_i(s) = \frac{1}{T_i}\frac{1-e^{-sT_i}}{s}, \quad i = 1, \ldots, n \tag{1}$$

where T_i is the time constant, so-called the delay of the *ith* FIR filter. Similarly, the FIR filter can be characterized by its rectangular impulse response with a unitary area:

$$m_i(t) = L^{-1}(M_i(s)) = \frac{u(t)-u(t-T_i)}{T_i}, \quad \text{where}\, u = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{2}$$

For the sake of simplicity, trajectory generation for a single-axis motion is considered, and a rectangular (pulse) velocity signal with magnitude of v is send to a FIR filter for a period of $T_v \geq T_i$. The filtered velocity signal can be computed through convolution:

$$\begin{aligned} \mathrm{v}'(t) &= \mathrm{v}(t) * m(t) = \frac{1}{T_i}\int_0^t ([\mathrm{v}(\tau) - \mathrm{v}(\tau - T_v)][u(t-\tau) - u(t - T_i - \tau)])\,d\tau \\ &= \frac{1}{T_i}\left| \begin{array}{l} \int_0^t \mathrm{v}(\tau)u(t-\tau)\,d\tau - \int_0^t \mathrm{v}(\tau)u(t-T_i-\tau)\,d\tau - \\ \int_0^t \mathrm{v}(\tau-T_v)u(t-\tau)\,d\tau - \int_0^t \mathrm{v}(\tau-T_v)u(t-T_i-\tau)\,d\tau \end{array} \right| \end{aligned} \tag{3}$$

The filtered velocity profile $\mathrm{v}'(t)$ is calculated by evaluating above integrals;

$$\mathrm{v}'(t) = \begin{cases} (\mathrm{v}/T_i)t, & 0 \leq t < T \\ \mathrm{v}, & T_i \leq t < T_\mathrm{v} \\ \mathrm{v}/T_i(-t + T_\mathrm{v} + T_i), & T_\mathrm{v} \leq t < T_v + T_i \\ 0, & T_v + T_i \leq t \end{cases} \tag{4}$$

As noted, FIR filter acts as a moving average and integrates the commanded velocity pulse. The resultant feed profile $\mathrm{v}'(t)$ becomes trapezoidal and acceleration is rectangular (see Fig. 2). Peak acceleration depends on the filter's time constant, $a_{max} = \mathrm{v}/T_i$. Due to the formal definition of convolution, total cycle time is sum of both time duration of the input velocity pulse and filter's time constant, $T_{total} = T_v + T_i$. Please note that T_v is computed based on the total travel length, L, and the commanded velocity,

$$T_v = \frac{L}{\mathrm{v}} \tag{5}$$

so that when convolved with the FIR filter, area underneath velocity pulse does not change, and thus integration of $\mathrm{v}'(t)$ satisfies desired position command, $s'(t)$.

Accordingly, higher order trajectories can be generated by applying series of n FIR filters. As shown in Fig. 2, if a second FIR filter is added to the structure, it convolves the rectangular acceleration function and converts it to first order. Maximum value of acceleration remains the same as a_{max}. Jerk profile becomes rectangular, and its maximum depends on the 2nd FIR filter's time constant, i.e. $j_{max} = a_{max}/T_2$. It should be noted that for this condition to work, there should be enough time for both of the filters to converge to steady state. In other words; $T_\mathrm{v} > T_1 > T_2$ should be satisfied. Total travel time is stretched by sum of the total delay in the chain,

$$T_d = T_1 + T_2 + \ldots T_n \tag{6}$$

2.2. Frequency shaping of reference trajectory

Proposed structure also allows us to shape frequency spectrum of the generated trajectory. Acceleration $a'(t)$ corresponds to the torque/force delivered by motors, which excites drive system,

$$\begin{aligned} a'(s) &= \mathrm{v}(s)sM_1(s)\ldots M_n(s) \to a'(j\omega) \\ &= \mathrm{v}(j\omega)(j\omega)M_1(j\omega)\ldots M_n(j\omega) \end{aligned} \tag{7}$$

Frequency response of the FIR filter can be evaluated from Eq. (1) as:

$$M_i(j\omega) = \frac{1}{T_i}\frac{1-e^{-j\omega T_i}}{j\omega} = e^{-j\omega T_i/2}\left(\sin\left(\frac{\omega T_i}{2}\right) / \frac{\omega T_i}{2}\right) \tag{8}$$

Overall spectrum of the acceleration is computed by product of individual FIR filters' frequency response as:

$$|a'(j\omega)| = \prod_{i=1}^{n}\left|\frac{\sin(\omega T_i/2)}{\omega T_i/2}\right| = \prod_{i=1}^{n}\left|\frac{\sin(\pi\omega/\omega_i)}{\pi\omega/\omega_i}\right|, \quad \text{where } \omega_i = \frac{2\pi}{T_i} \tag{9}$$

Above property can be exploited to choose parameters of the trajectory/filter in order to suppress lightly damped structural frequencies of the feed drive system or the servo itself. An example is presented in Fig. 3. For instance, in order to avoid excitation at a critical modal frequency ω_r delay of the filter is tuned to match one of the ripples of the FIR filter as:

$$\omega_i = \frac{\omega_r}{k} \leftrightarrow T_i = k\frac{2\pi}{\omega_r}, \quad k = 1, 2, \ldots, N \tag{10}$$

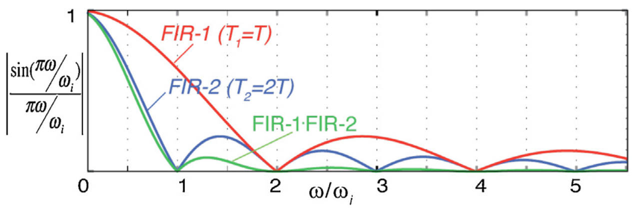

Fig. 3. Frequency response of FIR filters.

3. Accurate corner smoothing

As a G-line is processed, commanded feedrate is separated into axial velocity pulse commands and interpolated through FIR filters for each drive, which ensures synchronized multi-axis motion. This process stretches total cycle time of a G-line. Thus, interpolation of the successive G-line must be delayed so that a full stop is performed at their junction point. This is illustrated on a $\beta = 90^\circ$ corner trajectory shown in Fig. 4a. Feed motion is commanded on two 2.5 [mm] subsequent lines at a feedrate of $F = 50$ [mm/s]. This results in $T_v = 2.5/50 = 50$[ms] width of velocity pulses to be delivered. A single FIR filter, $T_1 = 20$ [ms] is utilized for the interpolation of drive commands, and total cycle time becomes $T_v + T_1 = 70$ [ms]. Corresponding acceleration and velocity profiles are shown in Fig. 4b. As noted, if successive G-lines are filtered with a delay of $T_d = 20$ [ms], convolution starts with zero initial conditions, full stop can be performed and hence sharp corners are machined. Please note that trapezoidal velocity profile is generated with a single FIR filter.

If consequent G-line is interpolated without fully waiting for the filter delay that is added to the previous one, it forces convolution to start with non-zero initial states, i.e. initial velocity, and acceleration. Here, the amount of overlap between successive G-line filtering is called the overlapping time, T_k

$$0 \le T_k \le T_d \tag{11}$$

which is utilized to control the cornering error. In Fig. 4c, T_k is set to $T_k = T_d$, and full-overlap is commanded. The successive block is interpolated immediately. As noted from Fig. 4c, in full-overlap X-axis does not slow down around the sharp corner where Y-axis reverses its velocity from $F\sin(45^\circ)$ to $-F\sin(45^\circ)$ with an maximum acceleration of $a_{max} = 2F\sin(45^\circ)/T_1 = 3535$ [mm/s^2]. Since both axes do not slow down before approaching the corner, the corner is smoothened with large contouring errors, and delivers the fastest cornering kinematics.

If the overlapping time is less, due to the moving average, both drives slow down for period of $T_d - T_k$ (see Fig. 4d), and then successive feed pulses enter the convolution to change the feed direction. Thus, drives slow down before approaching to the corner and alter their velocities to the succeeding feed vector. The case for $T_k = T_d/2 = 10$ [ms] is shown in Fig. 4d. Acceleration and velocity profiles can be computed by applying convolution with initial states. As an example, X-axis profiles become:

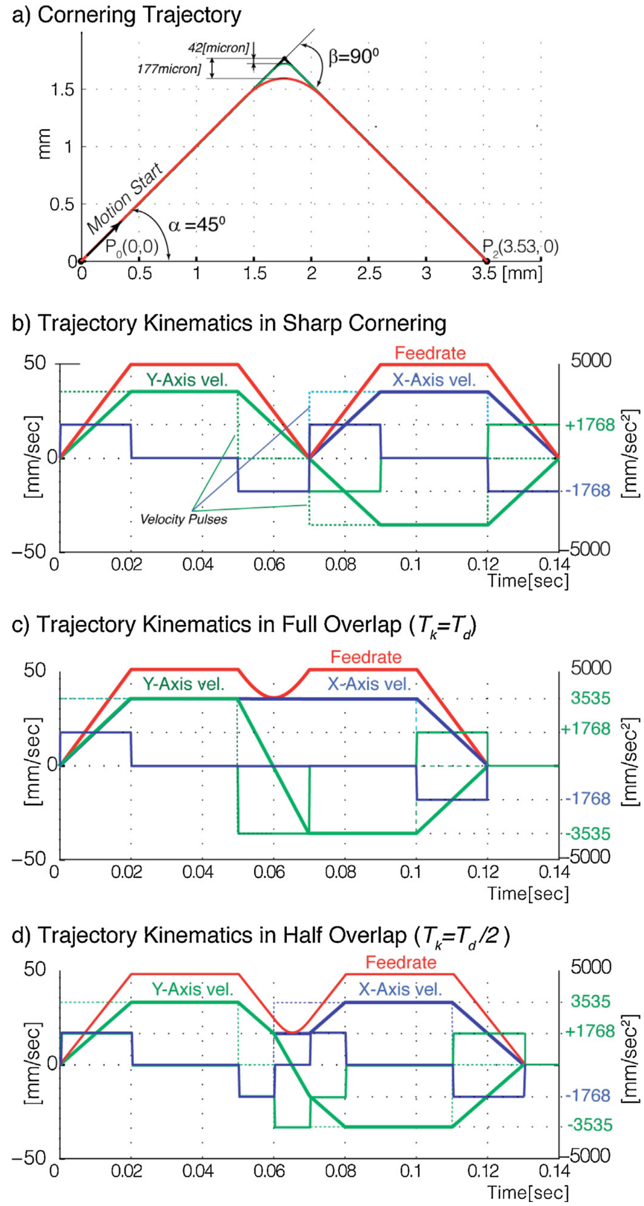

Fig. 4. Accurate corner blending based on G-line overlapping.

$$a_x^f = \begin{cases} \dfrac{-F_x^-}{T_1} \\ \dfrac{\Delta F_x}{T_1} \\ \dfrac{F_x^+}{T_1} \end{cases} \rightarrow \upsilon_x^f = \begin{cases} F_x^- - \dfrac{F_x^-}{T_1}t & , T_v \le t \le T_v + T_{slow} \\ F_x^- - \dfrac{F_x^-}{T_1}T_{slow} + \dfrac{\Delta F_x}{T_1}t & , T_v < t \le T_v + T_1 \\ F_x^- - \dfrac{F_x^-}{T_1}T_{slow} + \dfrac{\Delta F_x}{T_1}T_k + \dfrac{F_x^+}{T_1}t & , T_v + T_1 < t \le T_v + T_1 + T_{slow} \end{cases},$$
$$\text{where } \Delta F_x = F_x^+ - F_x^-,\ T_{slow} - T_1 - T_k \tag{12}$$

$F_x^- = F\cos(\alpha)$ and $F_x^+ = F\cos(\alpha - \beta)$ are X-axis velocity commands along successive G-lines. As long as F is constant, largest cornering error occurs in the middle of the overlapping time $T_v + T_1 - T_k/2$ due to symmetry. By integrating the velocity profile, axes positions and location of the largest cornering error can be calculated. In case of a single FIR, i.e. trapezoidal velocity, cornering error ε is calculated as a function of cornering angle, β and T_k as:

$$\varepsilon = \sqrt{\frac{F^2 T_k^4 (1 - \cos(\beta))}{32 T_1^2}} \tag{13}$$

For 2 FIR filters, the profile becomes jerk limited and for a given cornering error ε, necessary overlapping times are calculated as:

$$T_k = \sqrt[3]{\frac{24T_1T_2\varepsilon}{F\sin(\beta/2)}}, \quad 0 \le T_k \le 2T_2$$
$$T_k = T_2 + \sqrt{\frac{4T_1\varepsilon}{F\sin(\beta/2)} - \frac{T_2^2}{3}}, \quad 2T_2 \le T_k \le T_1 + T_2 \tag{14}$$

where β is the cornering angle. Consequently, by selecting the overlapping time between the blocks from Eq. (14) linear segments can be blended with the desired cornering tolerance ε set by the user. Owing to its simplicity, proposed Interpolation scheme is applied conveniently in real-time along entire G-code.

4. Experimental results

The proposed accurate cornering algorithm is experimentally tested on a two axis, ball-screw driven machine tool shown in Fig. 5. Cascade P-PI motion controller is implemented with inertia and friction feed-forward for accurate positioning. For demonstration, a pair of steel beams are placed on the X–Y table whose first bending modes are at $\omega_{n,X}$ = 11.4 [Hz] and $\omega_{n,Y}$ = 6.8 [Hz]. Frequency content of reference acceleration can easily excite those lightly damped low frequency modes. Time constants of the FIR filters in proposed method are selected as T_1 = 148 [ms] and T_2 = 89 [ms], respectively to avoid residual vibrations.

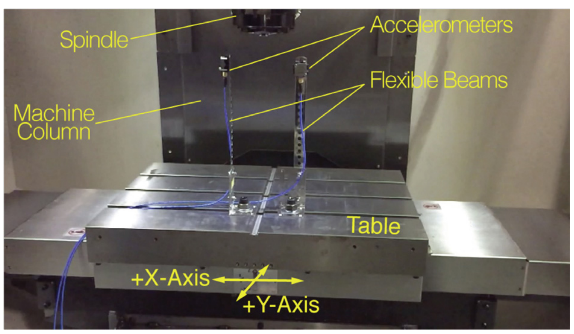

Fig. 5. Experimental setup.

A diamond shaped tool-path shown in Fig. 6a is commanded for continuous travel at a feedrate of F = 250 [mm/s], and the cornering tolerance is set to ε = 500 [micron]. The proposed cornering scheme is compared experimentally against the geometric corner-smoothing algorithm, which uses curvature optimal corner blending [4]. Fig. 6b shows feed profiles along the tool-path. As noted, proposed method delivers faster cycle time due to the fact that it can accelerate through the corner. In contrast, tool must cruise at a constant speed through the corners so that axis acceleration limits are respected in geometric smoothing method. The acceleration profiles of the axes cannot be controlled since they are dictated by the corner curvature and the feedrate. As a result, flexible beams are excited in each direction around the corner blends as shown in Fig. 6e–f. In the proposed method, corners are turned without stopping and without triggering any vibration on the beams (see Fig. 6f). Furthermore, by explicitly calculating the overlapping time with respect to the filter delay and the cornering angle, proposed method accurately rounds the corners with respect to given tolerances. The overall cycle time and motion accuracy is clearly better than the widely used geometric corner blending method.

5. Conclusions

A new trajectory generation scheme has been proposed for Cartesian machine tools for accurate cornering. As compared to conventional geometric path smoothing algorithms, the novel cornering scheme filters discrete tool-paths and rounds the sharp corners with respect to the desired tolerance while delivering the fastest cycle time. Utilizing chain of FIR filters, it guarantees high kinematic smoothness and at the same time avoids exciting lightly damped critical structural modes of the machine structure. The algorithm can be programed in real-time by utilizing moving average filters, and hence provides effective practical implementation for NC systems.

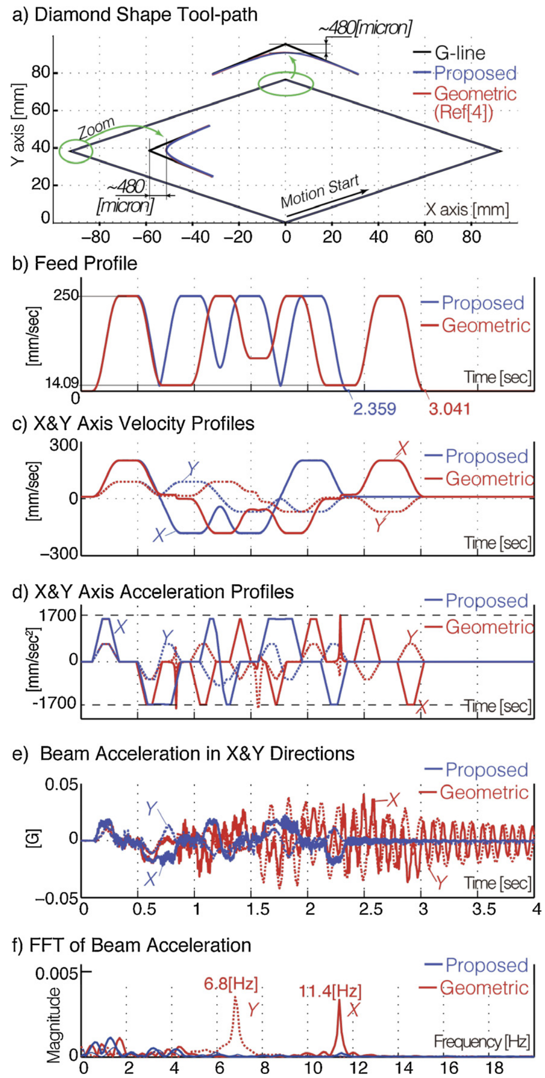

Fig. 6. Experimental results.

References

[1] Brecher C, Lange S, Merz M, Niehaus F, Wenzel C, Winterschladen M, Weck M (2006) NURBS Based Ultra-Precision Free-Form Machining. *Annals of the CIRP* 55(1):547–550.
[2] Koninckx B, Van Brussel H (2002) Real-Time NURBS Interpolator for Distributed Motion Control. *Annals of the CIRP* 51(1):315–318.
[3] Altintas Y, Erkorkmaz K (2003) Feedrate Optimization for Spline Interpolation in High Speed Machine Tools. *Annals of the CIRP* 52(1):297–302.
[4] Sencer B, Inagaki S, Shamoto E (2015) A Curvature Optimal Sharp Corner Smoothing Algorithm for High-speed Feed Motion Generation of NC Systems along Linear Tool-paths. *International Journal of Advanced Manufacturing Technology* 76(9–12):1977–1992.
[5] Yutkowitz SJ, Chester W, Apparatus and Method for Smooth Cornering in a Motion Control System, United States, Siemens Energy & Automation, Inc, Alpharetta, GA, 2005 (US Patent 6922606).
[6] Singhose WE, Seering WP, Singer NC (1996) Improving Repeatability of Coordinate Measuring Machines with Shaped Command Signals. *Precision Engineering* 18(2–3):138–146.
[7] Chen CS, Lee AC (1998) Design of acceleration/deceleration profiles in motion control based on digital FIR filters. *International Journal of Machine Tools and Manufacture* 38(7):799–825.
[8] Altintas Y, Khoshdarregi MR (2012) Contour Error Control of CNC Machine Tools with Vibration Avoidance. *Annals of the CIRP* 61(1):335–338.